To assess students’ attitudes, skills and competencies in mathematical modeling

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Abstract

Peer-to-peer assessment, take-home exams and a mathematical modeling survey were used to monitor and assess students’ attitudes, skills and competencies in mathematical modeling. The students were all in a secondary mathematics, teacher education program with a comprehensive amount of mathematics studies behind them. Findings indicate that peer-to-peer assessment is a fruitful way to introduce discussions about assessment of mathematical modeling, and of developing qualitative grading criteria for grading take-home exams. An internationally developed mathematical modeling survey was used as a final course-grading instrument.

I. Assessment and attitudes

To assess mathematical modeling is a much more complicated task than anticipated before one starts to teach mathematical modeling. The fact that the students will be assessed not only in the matter of whether the solution is right or wrong but also in logical reasoning and in their linguistic competency, calls for effort to be spent on explaining the assessment procedure for the students. Another important factor, with significant importance for what the student will learn during a course, is of course the previous knowledge of mathematics that the student has gained. And to that, we can also add the attitudes the students bring with them when they come to a course in mathematics. We consider it relevant to describe a person as having taken an approach to a task, since the use of approaches to learning is a good description of the relation between the learner and the object of learning within a particular context.

There is also research evidence that shows that students’ approaches to learning vary with context (1). Depending on the educational environment in different departments the student’s tendency to take a particular approach to learning varies from one institution to another. This is also stated by Prosser and Trigwell (2) as a fundamental relationship emanating from the study of student learning.

The learning approaches adopted by students are evoked by the relationship between the student and their perceptions of the context: they are not a characteristic of students. If the context is changed, there is the likelihood that the student’s approach will change (2, p. 92).

Equally interesting and important is the students’ view of mathematics. Crawford et al. (3) found that there are widely different ways to apprehend mathematics among first-year students.
at university. They claim that there is a borderline between those who comprehend mathematics as a ‘fragmented body of knowledge’ and those who have a ‘cohesive view of mathematics’ (p. 335). To conceive mathematics as a fragmented body of knowledge probably gives one the impression that mathematics mainly consists of rules and formulas, whereas a cohesive view of mathematics indicates that one sees mathematics as a complex way of thinking, of solving problems, and of gaining knowledge about the world. It is our perspective that mathematical modeling is an excellent way to learn more mathematics, but also to learn to appreciate many different aspects of mathematics. In what way do prospective mathematics teachers’ attitudes, mathematical writing and different assessment strategies influence the learning outcome in a specific modeling course?

2. Peer-to-peer assessment

The value of peer review in higher education is widely recognized by educators and educational researchers. There are researchers who claim that a student’s work will be of better quality if his or her peers are involved in the assessing procedure. Peer review is also a way to move the focus away from a system with an ‘expert’ instructing ‘novices’ through teaching. This in turn will hopefully encourage students to become more actively involved in their educational enterprise:

It’s worth emphasizing that it is not always necessary for academic staff to give feedback: students can often learn more from formal or informal assessment by their peers or by themselves (1, p. 195).

Houston and Lazenbatt (4) found that extensive and focused group work with emphasis on a group’s responsibility for delivering a solution to a mathematical modeling problem created a positive learning atmosphere, especially in developing group work skills and personal skills of communication and presentation. The more responsibility one gives to a group, the more informal and formal assessment will take place within the group.

Peer review includes many qualities from a learning perspective and in the research community peer review is probably the most widely used approach when evaluating research. When writing a report on a mathematical modeling exercise, students are likely to benefit most from a shared instructor and peer review where the criteria for evaluating, marking, and eventually grading, is discussed from a mutual platform. We normally do this in a seminar with our student teachers. The prospective teachers will get a concise modeling exercise, which they are expected to hand in after some 10 days. In our setting, they deliver two copies, one for the instructors and one for someone in their peer group. One peer and one instructor mark every student’s work. Finally, the marking system and suitable criteria for grading is discussed in a seminar where there is time and room for students to share and express their attitudes regarding this way of working with mathematical modeling. They are also asked to answer a survey that more specifically goes into the peer review exercise, something most of the students are unfamiliar with.

Some of the main uses of peer assessment are to help students to deepen their understanding of a topic or a method and the processes of assessment and to facilitate the development of reflective learning in mathematical modeling. With the aim of involving the students in the processes of assessing work they have done in the modeling exercise there was an ongoing discussion about criteria. This is a way to reach understanding and concordance among the students in the group.
The criteria for peer marking may be developed by the groups of students involved. This approach develops understanding and a sense of ownership of the criteria (5).

One important part of the discussion with students is naturally about relevant criteria to use when marking and grading. Our experience is that the students are quite capable of inventing a well thought-out list if they are just given the opportunity. The students in this study invented the following summarized list:

- Is the overall question answered? Is the text easy to follow and well structured? Are the sources presented properly? Is there a ‘common thread’?
- Is the report clear and distinct? Is the idea behind the text easy to follow? Is the argumentation correct? Are figures and tables handled properly? Are mathematical concepts treated correctly?
- The peer review also contained collegial difficulties, where some tasks were easier than others to handle in the peer review (see Table 1).

### Table 1

<table>
<thead>
<tr>
<th>Easier</th>
<th>Harder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factual errors</td>
<td>Relation to the author</td>
</tr>
<tr>
<td>Wrong answer</td>
<td>Unfamiliarity with criteria for marking</td>
</tr>
<tr>
<td>Mark text</td>
<td>Language – form – content</td>
</tr>
<tr>
<td>Common language</td>
<td>Mathematical language</td>
</tr>
<tr>
<td>Easier when it corresponds to my own work</td>
<td>When the report is sparsely written</td>
</tr>
<tr>
<td></td>
<td>To formulate constructive criticism—what should be criticized</td>
</tr>
</tbody>
</table>

### 3. The take-home exam

After the peer-review exercise, the students are considered prepared to take on three different modeling problems in a take-home exam. The three problems are generally divided into one dynamical-geometry modeling problem and two more ‘common’ modeling problems. These problems naturally alter for every semester the course is offered and a variety of some of the different problems used in these courses by the authors can be found in Holmquist and Lingefjärd (6), Lingefjärd and Holmquist (7,8), Lingefjärd (9–11) and Lingefjärd and Kilpatrick (12). The peer-review exercise is expected to have caused the students to focus on different aspects of writing mathematics.

For the last 10 or 15 years there has been a growing concern about how to create and implement relevant writing assignments throughout the mathematics curricula. Different reasons have contributed to a movement towards the use of writing assignments in mathematics. These include the ready availability of increasingly advanced technology to learners of mathematics, a growing interest among teachers of mathematics at all levels to learn more about what and how their students learn, and a likewise growing certainty among researchers in mathematics education that assessing knowledge in mathematics requires much more than just a written test at the end of a course.

Many authors have discussed the need and importance of students’ ability to communicate mathematics orally and in writing (13–15). Another ongoing discussion regarding the need for students to take responsibility for their own learning often connects this aim with empowering
the students to read, write and discuss mathematics intelligently (6–12,16). A third argument is that it is a realistic view of where students will continue with professional careers after their education. Many of them will most likely enter into fields where they will be performing scientific, technical, or economics reading and writing, and thus they will benefit from writing mathematics in their educational program.

In general, it is not easy to convince either teachers or prospective teachers of mathematics that they should be part of the writing procedures in the same way as, for instance, teachers of science, where students write essays, lab reports, and so forth. Common reactions are:

That is not part of my duty: I’m a mathematician, not a writing instructor!
For that part, how do you grade a report or an essay?
Where do I find good writing assignments for my mathematical course?
[developed from Meier & Rishel (13), p. ix].

A reluctance to set written assignments is probably connected to the view one has about assessment and student performance. Is a controlled written paper-and-pencil test the only way to assess students or do you believe in the statement that there are at least three basic sources of assessment information from students: observations, their responses to questions, and a coherent examinations of their work? The communicative aspect of learning mathematics is found in the Principles and Standards for school mathematics (17) as a strand for all grades. It concludes that:

Writing is a valuable way of reflecting on and solidifying what one knows, and several kinds of exercises can serve this purpose. For example, teachers can ask students to write down what they have learned about a particular topic or to put together a study guide for a student who was absent and needs to know what is important about the topic. Students who have done a major project or worked on a substantial long-range problem can be asked to compare some of their early work with later work and explain how the later work reflects greater understanding. In these ways, teachers can help students develop skills in mathematical communication that will serve them well both inside and outside the classroom. Using these skills will in turn help students to develop deeper understandings of the mathematical ideas about which they speak, hear, read, and write [National Council of Teachers of Mathematics (17), p. 352].

One such mathematical modeling problem that encouraged our students to accomplish a lot of writing was the following task, given in the take-home exam:

**4. Modeling drug responses**

L-Dopa is administered to people suffering from Parkinson’s disease to relieve symptoms such as extreme tremors and rigidity. Consider the set of data given in Table 2 on the levels of L-dopa

| Table 2 Levels of L-dopa in the blood after drug administration |
|-------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| t (min)           | 0      | 20     | 40     | 60     | 80     | 100    | 120    | 140    | 160    | 180    | 200    | 220    | 240    | 300    | 360    |
| L-dopa (ng/ml)    | 0      | 300    | 2700   | 2950   | 2600   | 1550   | 1100   | 900    | 725    | 600    | 510    | 440    | 300    | 250    | 225    |
in the blood, in nanograms per milliliter, as a function of time, in minutes after the drug is administered.

a. What function would you use to fit to these data points? Give at least three different suggestions and discuss the validity and reliability of each of the three functions you propose.

b. When is the level of L-dopa in the blood lower than 10 ng/ml? Relate your answers to each of the three models in (a).

c. How do you interpret the area under the curves of your function? Calculate the area in all three cases and give a good explanation of how you interpret the meaning of measuring the area and how you understand the result.

It is our experience that a problem like the L-dopa problem cannot be solved and presented without the students becoming involved in a lot of conjecturing, discussion and mathematical writing.

5. The attitude survey
In order to understand whether the peer-review exercise affected the students’ attitudes about mathematics, the teaching and learning situation, technology, marking and grading exercises, and so forth, the students were asked to complete a questionnaire with 32 different questions divided into several groups. The subtitles addressed the areas:

- What have you learnt about mathematics?
- What have you learnt about working methods and ways of organizing the work?
- What have you learnt about the use of technology with respect to the teaching and learning of mathematics?
- In what way has the course affected your view of mathematical modeling?
- What are your opinions about marking a peer’s paper?
- What are your opinions about being evaluated by a peer?
- What are your opinions about the course with respect to demands and accomplishment?

It turned out that about one third of all the students did not approve of the peer-review process, they did not enjoy being evaluated on their written performance, and did not like the way the course was organized as a whole. The majority of the students did, however, approve, and what was even more positive is that the same majority considered it highly important to be evaluated by peers and to be introduced to self-evaluation. Another finding from this survey was that the overall majority of the students concluded that teaching, learning and assessing mathematical modeling was much harder and more complicated compared to what they had anticipated before taking the course.

6. Students’ skills and competencies
A group of researchers in mathematics and mathematics education, who are spread around the world in Australia, England and Ireland, and are concerned about how to detect and recognize students modeling achievement, have devised assessment strategies and a mathematical-modeling test for measuring general and specific competencies in modeling and applications.
The mathematical-modeling test used in this study is intended to collect evidence of growth in mathematical modeling competencies (18). Reports of research about these assessment approaches and procedures have been presented at ICTMA conferences since 1991. For more information about the ICTMA community, and how to obtain a huge variety of examples of applied mathematics, please see http://www.infj.ulst.ac.uk/ictma/.

Prospective teachers need to understand a great variety of topics and approaches in mathematics. Today these topics include concepts, principles, methods and procedures inside mathematics and inside many engineering subjects. Applied mathematics as a field and the process of mathematical modeling in particular are parts of the mathematical curriculum for prospective teachers that may be broadened, enhanced and become even more important in the future because of the continuous technological revolution.

7. Assessment validity

The concept of test validity for an assessment instrument is only an objective measure and evidence that the test actually measures what it purports to measure. In order to call student assessments valid, we should ask for a comprehensive list of fulfillment. The assessment procedures should provide quality assurance for certification of achievement or professional recognition, for informing management, and for evaluation of innovations and developmental interventions. Students' assessment is a powerful influence on what is taught and what will be taught and reshapes our notion of what is important in the educational program. It also gives students a better understanding of what is expected of them in a program, and in their future professions. Finally, it gives program coordinators and people in superior positions, a good indication of what students actually know (in a more general sense) after completion of their programs.

The concept of validity refers, in this context, to the extent to which meaningful, appropriate and useful inferences can be made about students' responses on a test used for particular purposes. Evidence of validity for a purpose can be construct-related, content-related and criteria-related (both concurrent and predictive evidence). The main validity issue is the extent to which meaningful, appropriate and useful inference can be made about assessment scores.

The mathematical-modeling test was completed in March 2004, when the students were in their second year of studies in mathematics and had just finished a course in mathematical modeling and in mathematical statistics. At this time, after at least two years of studies in their teacher program, the students had studied a variety of courses that addressed the topics in the mathematical-modeling test. The allotted test time was 60 minutes and all students answered all problems.

A total of 20 students participated and attempted to answer the 22 questions in the mathematical-modeling test (maximum score is 44). Nine were women, and 11 were men. The mean value for the whole group (n = 20) was 23.5 with a standard deviation equal to 5.7. As seen in Fig. 1, there were three students who scored 30, one who scored 32, and three students who scored 12, 14 and 17, respectively.

There were no significant differences between women and men in the results, but it is notable that the overall result from the test is satisfactory in comparison with other comparable groups recently taking the test. The most difficult questions for the 20 respondents were question number 5 (solving frequency = 20%), closely followed by question number 17. The question the students got the best score on was question number 20, with a solving frequency of 83%.
Multiple-choice tests are normally used for large samples of students because they are easy and quick for the students to respond to and often easy to mark. Every multiple-choice test is built around one introductory question or incomplete statement at the beginning of each item, which is followed by several options. The options consist of the answer (the correct option) and several distracters (the incorrect but, in general, equally tempting options). A multiple-choice test needs to balance the correct and the incorrect answers in such a way that the respondents hopefully experience all options as equal likely.

To write a multiple choice test that takes into consideration all different aspects of mathematical modeling is of course a truly hard task. The test used in this survey consists of eight different phases in the process of mathematical modeling; each is defined in Table 3 and is exemplified by a specific question in the test.

Every one of the 22 different problems has one correct answer, worth 2 score points, and at least one ‘almost’ or ‘nearly’ correct answer worth 1 point. It means that the maximum score, when students answer all problems correctly, is 44 score points.

When using a multiple-choice test with five different possibly-correct answers, one must check what the probability is that one could guess a given number of correct answers. This falls under the binomial probability theorem. It is easy to conclude that the probability to guess 10 correct answers is about 0.00455. Obviously we do not need to worry much about the probability that someone will score well on the survey by simply guessing, for instance, the correct answers of 10 questions.

**Problem 5.** Consider the real world problem (do not try to solve it!):

*What is the best size for pushchair wheels?*

Which **one** of the following clarifying questions most addresses the smoothness of the ride as felt by the child?

A. Does the pushchair have three or four wheels?
B. What is the distance between the front and the back wheels?
C. Is the seat padded?
D. How old is the child?
E. Is the pavement tarmac or paving slabs?
Problem 17. Which one of the following options most closely models the speed of a car starting from rest (in terms of time $t$)?

A. $1 - e^{-t}$  
B. $(1 - t)^2$  
C. $t$  
D. $t - t^2$  
E. $1/(1 + e^{-t})$

Problems 5 and 17 may be found in Haines et al. (19).

Problem 20. The following situation has been partially modeled:

An aircraft is waiting to land at a busy airport. It has been stacked at a constant height flying on an approximately circular path at a fixed speed. At a particular moment the aircraft is instructed to land and to taxi some distance to the airport terminal.

Which one of the graphs shown in Fig. 2 best represents the variation in the speed of the aircraft as the distance covered increases, from the stacking situation to the arrival at the terminal?

Problem 20 may be found in Houston and Neill (20).

One result from the mathematical-modeling test was that the students seemed to handle problems with some sort of visualization much better than problems with just text or symbols. None of the students expressed an explicit opinion concerning this, but the solving frequencies were much higher for test items either with a diagram or where a diagram could easily be envisioned. In problem 5 there was probably a linguistic difficulty, since the test was given in English and the phrase ‘Is the pavement tarmac or paving slabs?’ is not easily understood by Swedish students.

8. Discussion

Our conclusions are obviously drawn from a wide range of sources. Our data have been collected from seminars notes, discussion documents and take-home exam papers, where the students’ views of the process of mathematical modeling were in focus. We have observed and analyzed students’ learning outcomes with respect to different phases in the process. The students have
been involved in peer review work, and in this manner we have tried to understand how attitudes, mathematical writing and different assessment strategies influence the learning outcomes of a certain mathematical modeling course in a teacher education program. We consider that we have found clear evidences for the fact that attitude, mathematical writing and assessment strategies have a major impact on how students, over time, develop skills and competencies in mathematical modeling. A way to illustrate the overall structure is shown in Fig. 3.

It is not assumed that the relations between the boxes representing ‘attitudes’, ‘writing’ and ‘assessment strategies’ are fixed, but that there are dynamic relations.

Change is naturally hard for many prospective teachers in confronting a course in mathematical modeling such as the one studied. They are in a baffling, difficult situation. They come from many years of mathematical studies in which the primary teaching methodology has called for the reproduction of knowledge, and in which there is a clear structure consisting of textbook, instructions for studying, and regular lectures that illustrate the content of the textbook and of the course. And at the end of the course there are often previous examination papers to use

Fig 2. Graphs describing the variation in the speed of the aircraft as the distance covered increases.
for practice. Their view of mathematics and of studies in mathematics as having a fixed character has been well established and is difficult to shake.

Nevertheless, to engage students in evaluation of their peers’ work in order to encourage them to be more careful in the way they write about and argue for their own mathematical models, tends to be a fruitful way to bring about some change in the students’ views of mathematics, of mathematical modeling and of their view of teaching and learning mathematics. We encourage other teachers of mathematics to try out peer-review exercises and encourage students to define criteria for course assessment. It is likely to create a better learning environment in most courses.

References


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